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SATURATION CURRENTS INTO A PROBE IN A DENSE PLASMA

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In connection with intensive studies concerning the flow of an ionized gas, there has now also developed a considerable interest, according to the literature on this subject, in electric probes widely used as a major diagnostic tool. Electric probes are rather simple devices, but difficulties arise in the interpretation of a measured current—voltage characteristic and, especially, when such a probe operates in the hydrodynamic mode, i.e., when the mean free path of particles is much shorter than the characteristic probe dimension and the thickness of the Debye layer. In this study we will theoretically analyze the trend of the current—voltage characteristics of single electric probes in streams of dense plasmas at high positive and negative surface potentials when $\varepsilon = \lambda_d^2/L^2 \ll 1$ (λ_d denoting the characteristic quiescent Debye radius and L denoting the characteristic scale of change in the hydrodynamic parameters near the probe surface). This problem was for the first time considered in [1], where an asymptotic analysis of it at the limit $\lambda_d^2/\delta^2 \to 0$ (δ denoting the thickness of the viscous boundary layer) has led to the conclusion that the current—voltage characteristic of a probe at high positive (or negative) surface potentials levels off to constant values corresponding to the electron (ion) saturation current. Explicit expressions have been derived for the density of saturation currents which involve the normal derivative of the quasineutral concentration of charged particles at the wall. However, the assumptions made in that analysis greatly limit the applicability of the obtained results and, generally, make them unsuitable for direct diagnosis. It has been assumed in [1], e.g., that the gas temperature and density as well as the transfer coefficients are uniform in space, which implies equal temperatures of the probe surface and the unperturbed stream. In most experiments this condition is not fulfilled, however, since the probe is usually much colder than the plasma unperturbed by it. Collisions between charged particles as well as gaseous-phase ionization and recombination processes have also been disregarded in that study, which is not permissible in the interpretation, e.g., of probe measurements in the plasma of open-cycle MHD generators. It has been, furthermore, assumed in [1] that the gas flows in the boundary-layer mode and, accordingly, it is not possible to interpret, for instance, the probe measurements in a slowly moving plasma of the flame type [2] or the probe measurements under conditions of supersonic streamlining of blunt bodies by weakly charged plasmas in the "viscous shock lay-

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er" mode [3]. Neither has been considered the problem of saturation currents in the case of a collision-free Debye layer (this problem becomes significant when the concentration of charged particles is increased). The expressions for saturation currents derived in [1] have so far not been used in the technical literature for the derivation of specific diagnostic formulas. In this study the problem of saturation currents will be treated with the variability of gas properties, the gaseous-phase ionization and recombination processes, and also the collisions between charged particles taken into account. The analysis will be shown to remain valid also in the case of collision-free Debye layer. With the resulting general expressions for the density of saturation currents, diagnostic formulas can be derived for a large class of specific gas flow patterns under thermal equilibrium. Such a derivation was begun in an earlier study [4] by determining the distribution of density of the ion saturation current over the frontal surface of a sphere streamlined by a supersonic plasma stream in the "viscous shock layer" mode. It ought to be noted here that the system of equations in the hydrodynamic theory of electric probes in a plasma [5] is a nonlinear one with the coefficient of the highest-order derivative being a small parameter and that it cannot be solved analytically. There is an efficient method available [6] for numerically solving one-dimensional problems in the hydrodynamic theory of probes, but multidimensional problems of this kind are still difficult to solve numerically. An exact calculation of the current-voltage characteristic is, for this reason, impossible in most cases. In order to determine the saturation current, on the other hand, it is sufficient to calculte only the quasineutral concentration of charged particles and for this there are available well-developed methods including analytical ones. In such situations, evidently, the proposed approach is in many cases the only asymptotically exact one which will yield specific diagnostic tools.

1. We consider an ionized gas of three components (single-charge ions of one kind, electrons, and one kind of neutral particles) streamlining a charged conductor (electric probe). The mean free path is assumed to be small relative to the characteristic dimension of the streamlined body and the degree of ionization of the gas to be moderate, i.e., sufficiently low for the flow of the neutral component to remain unaffected by the presence of charged particles. This means that the degree of ionization must not exceed a few percent. Then the fields of all quantities pertaining to the gas as a whole, namely of its mean flow rate as well as its density and temperature, can be calculated from the solution to the corresponding streamlining problem without taking into account ionization. In our setup, therefore, these fields will be regarded as given. The problem is thus to determine both electron and ion concentrations and diffusion currents as well as the electric potential.

We first assume that the mean free path greatly exceeds the thickness of the Debye layer at the streamlined body. Then as the system of hydrodynamic equations governing over the entire volume occupied by the gas (conservation and transport of ions and electrons) we can use the Stefan-Maxwell relations [7] and the Poisson equations

$$\rho\mathbf{v}\nabla c_j + \operatorname{div}\mathbf{J}_j = \dot{w}_j \ (j = i, e); \tag{1.1}$$

$$\nabla x_j = \sum_{k=i,e,n} x_j x_k a_{jk}\left(\frac{\mathbf{J}_k}{\rho_k} - \frac{\mathbf{J}_j}{\rho_j}\right) - \frac{x_j}{kT} e_j \nabla\varphi + x_j\left(\frac{m_j}{m_n} - 1\right) \tag{1.2}$$

$$\times \nabla \ln p + x_j \textstyle\sum_j^{\mathrm{T}} \nabla \ln T \quad (j = i, e);$$

$$\Delta\varphi = -4\pi ne(x_i - x_e), \tag{1.3}$$

$$x_j = \frac{n_j}{n}, \quad c_j = \frac{m_j}{m_n} x_j, \quad \rho_j = m_j n x_j,$$

$$\mathbf{J}_n = -\mathbf{J}_i - \mathbf{J}_e, \quad \rho_n = \rho, \quad x_n = 1,$$

$$\sum\nolimits_j^{\mathrm{T}} = \sum_{k=i,e,n} x_k a_{kj}\left(\frac{D_k^{\mathrm{T}}}{\rho_k} - \frac{D_j^{\mathrm{T}}}{\rho_j}\right),$$

where v, ρ, T, p, n are the mean-mass velocity, the density, the temperature, the pressure, and the numerical concentration of the gas; x_j, c_j, n_j are the molar, the relative (mass), and the numerical concentration of the j-th component; ρ_j, J_j, $\dot{w}_j$ are the density, the mass-diffusion current, and the mass increment of the j-th component per unit volume and per unit time due to chemical reactions; e_j, m_j are the charge and the mass of a particle of the j-th kind; a_{ij}, D_j^T are the drag coefficients (in the first approximation reciprocals of the binary-diffusion coefficients) and the thermodiffusion coefficients; e is the charge of an electron; k is the Boltzmann constant; and subscripts i, e, n refer respectively to ions, electrons, and neutral particles.

The equations of transport have been written on the assumption that the temperature of electrons is in equilibrium with the temperature of heavy particles.

The transport coefficients a_{jk}, Σ_j^T can be calculated according to the kinetic theory of gases and will in this setup be regarded as given functions of the gas density and temperature as well as of, generally, the concentrations of charged particles.

We further assume that the ratios of drag coefficients, ion to neutral particles and electron to neutral particles, remain constant in the stream. This assumption is one of the most essential ones for this analysis. The accuracy with which this assumed condition holds true depends on the model used for describing the interactions of ions with neutral particles and electrons with neutral particles. It should be pointed out that in the most customary models of solid spheres and Maxwell molecules [5], in the first approximation, this condition holds true exactly when the perturbation part of the distribution function is expanded into Sonin polynomials.

Let us now formulate the system of boundary conditions. At the streamlined surface, which we assume to be ideally absorbing and nonemitting as well as ideally catalytic, the boundary conditions for ion and electron concentrations follow from the condition of mass balance [8], these conditions being stipulated so as to make both the ion concentration and the electron concentration at the streamlined surface zero for small values of the Knudsen number. The surface potential is assumed to be given.

Far from the streamlined body the concentrations of ions and electrons approach their levels in an unperturbed gas, while the electric potential tends to zero.

We thus have the boundary conditions:

at the streamlined surface

$$x_i = x_e = 0, \ \varphi = \varphi_w \ \text{(given)}; \tag{1.4}$$

far from the streamlined body

$$x_i \to x_\infty, \ x_e \to x_\infty, \ \varphi \to 0. \tag{1.5}$$

We now introduce the dimensionless variables

$$\tilde{\rho} = \frac{\rho}{\rho_0}, \quad \tilde{\mathbf{v}} = \frac{\mathbf{v}}{v_0}, \quad \tilde{\nabla} = L\nabla, \quad \tilde{\mathbf{J}}_j = \frac{La_0 m_n}{m_j \rho_0}\mathbf{J}_j, \quad \tilde{w} = \frac{\dot{w}_i}{\dot{w}_{i0}} = \frac{\dot{w}_e m_i}{\dot{w}_{i0} m_e}, \quad \tilde{a}_{kj} = \frac{a_{kj}}{a_0}, \quad \tilde{\varphi} = \frac{e\varphi}{kT_0}, \quad z_j = \frac{e_j}{e}, \quad \tilde{T} = \frac{T}{T_0}.$$

Subscript "0" denotes here the corresponding characteristic values.

The system of equations (1.1)-(1.3) becomes, in these new variables,

$$\mathrm{Re}\, \rho \mathbf{v} \nabla x_j + \operatorname{div} \mathbf{J}_j = D\dot{w} \ (j = i, e); \tag{1.6}$$

$$\nabla x_j = \sum_{k=i,e,n} \frac{x_j x_k}{\rho} a_{jk} \left(\frac{\mathbf{J}_k}{x_k} - \frac{\mathbf{J}_j}{x_j} \right) - \frac{x_j}{T} z_j \nabla\varphi + x_j \left(\frac{m_j}{m_n} - 1 \right) \nabla \ln p + x_j \Sigma_j^T \nabla \ln T \quad (j = i, e); \tag{1.7}$$

$$\mathbf{J}_n = -\frac{m_i}{m_n}\mathbf{J}_i - \frac{m_e}{m_n}\mathbf{J}_e; \tag{1.8}$$

$$\varepsilon \Delta\varphi = -\rho (x_i - x_e), \tag{1.9}$$

where $N_{Re} = a_0 v_0 L$; $D = \dot{w}_{i0} m_n a_0 L^2/\rho_0 m_i$; $\varepsilon = kT_0 m_n / 4\pi\rho_0 e^2 L^2$ (the tilde above dimensionless variables has been omitted). The form of boundary conditions (1.4) and (1.5) remains unaffected by the introduction of new variables.

The quantity ε will be regarded as the small parameter in the problem. The entire flow field can then be split into a quasineutral region and a Debye boundary layer with a strong separation of charges [1, 9] adjacent to the streamlined surface.

2. The object of this study is the trend of ion and electron characteristics, i.e., the dependence of ion and electron currents on the surface area and the electric potential of the probe in the range of high positive and negative potentials. Let I_j denote the dimensionless density of the current referring to particles of the j-th kind and flowing to the streamlined surface, namely

$$I_j = -J_{jyw} \; (j = i, e).$$

Here and henceforth index "w" refers to the values of respective quantities at the streamlined surface and the y axis runs normally to that surface.

One should expect, on the basis of physical considerations, that the ion (electron) current to the streamlined surface decreases to zero when the electric potential rises (drops) to infinity:

$$\text{at } \varphi_w \to \infty \; I_i \to 0; \qquad \text{at } \varphi_w \to -\infty \; I_e \to 0. \tag{2.1}$$

The problem is to determine the trend of the electron characteristic at high positive surface potentials and of the ion characteristic at high negative surface potentials.

3. We will first consider the case of weak ionization, i.e., assume that collisions between charged particles occur at a much lower frequency than their collisions with neutral particles. Inasmuch as the cross section for collisions between charged particles is several orders of magnitude larger than the cross section for their collisions with neutral particles, the degree of ionization must not exceed 10^{-4}-10^{-5}. In the transport equations (1.7) for ions, e.g., one ought to omit the terms in the first part on the right-hand side which represent collisions with electrons. It is also simpler now to calculate the transport coefficients Σ_j^T, a_{jn} (j = i, e), namely, just as in the case of a binary mixture consisting of particles of the j-th kind and neutral ones. It will be noted that these transport coefficients depend on the gas density and temperature but not on the concentration of charged particles.

The Stefan-Maxwell relations (1.7) become in this case the generalized Fick's laws

$$\mathbf{J}_j = \frac{\rho}{a_{jn}}\left[-\nabla x_j - \frac{x_j}{T} z_j \nabla\varphi + x_j\left(\frac{m_j}{m_n} - 1\right)\nabla \ln p + x_j \sum\nolimits_j^{\mathrm{T}} \nabla \ln T\right]; \tag{3.1}$$

$$\sum\nolimits_j^{\mathrm{T}} = -\frac{D_j^{\mathrm{T}}}{\rho_j} a_0 a_{jn} = -\frac{K_{\mathrm{T}j}}{x_j} \quad (j = i, e), \tag{3.2}$$

where K_{Tj} is the thermodiffusion ratio for a binary mixture consisting of particles of the j-th kind and neutral ones [10], a_{jn}^{-1} is the coefficient of binary diffusion for this mixture, and Σ_j^T is equal, except for the sign, to its thermodiffusion coefficient [10]. In this particular approximation charged particles interact only through the electric field.

Let us examine the quasineutral system of equations describing the first term of the external asymptotic expansion of the solution to problem (1.4)-(1.6), (1.9), (3.1). This system consists of Eqs. (1.6), (3.1), and the degenerate Poisson equation (1.9), i.e., the condition of quasineutrality

$$x_i = x_e = x. \tag{3.3}$$

Combining relations (3.1), with this condition taken into account, yields

$$a_{in}\mathbf{J}_i + a_{en}\mathbf{J}_e = \rho\left[-2\nabla x + x\left(\frac{m_i + m_e}{m_n} - 2\right)\nabla \ln p + x\left(\sum\nolimits_i^{\mathrm{T}} + \sum\nolimits_e^{\mathrm{T}}\right)\nabla \ln T\right]. \tag{3.4}$$

Multiplying Eqs. (1.6) by $a_{jn}/(a_{in} + a_{en})$ and then adding the resulting equations yields the equation

$$\mathrm{Re}\,\rho \mathbf{v}\nabla x + \operatorname{div} \rho D_a\left[-\nabla x + x\left(\frac{m_i + m_e}{2m_n} - 1\right)\nabla \ln p + x\frac{\sum_i^{\mathrm{T}} + \sum_e^{\mathrm{T}}}{2}\nabla \ln T\right] = D\dot{w}. \tag{3.5}$$

for the concentration of charged particles in the quasineutral region, where $D_a = 2/(a_{in} + a_{en})$ is the coefficient of ambipolar diffusion.

The boundary condition for this equation far from the streamlined body is identical to the first of conditions (1.5), namely

$$x \to x_\infty. \tag{3.6}$$

The boundary condition for the quasineutral concentration at the streamlined surface is established by collocating with the expansion of the solution for the Debye layer. In the general case this layer cannot be described within the framework of a single asymptotic expansion but consists of several boundary layers, each describable by such a corresponding individual expansion [11]. Within the transitional boundary layer, which happens to be the "outermost" one in the sense that its expansion collocates directly with the quasineutral expansion, the concentration of charged particles expands into a series beginning with terms proportional to $\varepsilon^{1/3}$. Obviously, the condition of collocation here is that the first term in the outer asymptotic expansion of the concentration become zero at the streamlined surface:

$$x = 0 \text{ at } y = 0. \tag{3.7}$$

It is important to note that the distribution of charged particles according to Eq. (3.5) with the boundary conditions (3.6) and (3.7) does not, in the quasineutral approximation, depend on the electric potential of the streamlined surface.

The normal to the streamlined surface components of ion and electron diffusion currents across the Debye layer, which is assumed here to be asymptotically thin, are constant in the first approximation and equal to those components at y = 0 in the quasineutral approximation. Consequently, ion and electron currents flowing to the streamlined surface are constrained according to relation (3.4) written in a projection on the y axis at y = 0:

$$a_{in} I_i + a_{en} I_e = 2\rho \partial x / \partial y.$$

From this relation and conditions (2.1) we find the electron current at high positive surface potentials and the ion current at high negative surface potentials

$$I_e = \frac{2\rho}{a_{en}} \frac{\partial x}{\partial y}\bigg|_{y=0}; \tag{3.8}$$

$$I_i = \frac{2\rho}{a_{in}} \frac{\partial x}{\partial y}\bigg|_{y=0}. \tag{3.9}$$

Inasmuch as function x does not depend on the electric potential of the streamlined surface, according to our assumptions, these expressions describe the density of saturation currents. When $\rho = 1$, then these expressions become those in [1] for a chemically "frozen" gas with constant properties flowing in the "boundary layer" mode.

The results are illustrated in Fig. 1 by exact electron characteristics of a plane electric probe in a quiescent homogeneous gas [6]. As the value of parameter ε decreases, these characteristics approach the $2/a_{en}$ level at high potentials and this also corresponds exactly to relation (3.8).

We note that Eqs. (3.8) and (3.9) have been derived from the first terms in the outer asymptotic expansion and thus are accurate down to the second terms. The order of magnitude of the second terms in the outer asymptotic expansion is determined by the thickness of the Debye layer and strongly depends on the electric potential applied to the streamlined surface. At very high potentials ($\varphi_w \approx \varepsilon^{-1/2}$), for instance, the thickness of the Debye layer approaches the order of unity [11]. Our analysis, therefore, applies only to the range of rather low electric potentials of the streamlined surface and correspondingly small thicknesses of the Debye layer.

As the value of parameter ε decreases at a sufficiently high but not fixed electric potential, the thickness of the Debye layer decreases and the current density approaches the value stipulated by (3.8) or (3.9). This trend is clearly seen on the diagram in Fig. 1.

It is also noteworthy that at a very high applied potential the electric field near the surface will be strong and its work through a distance equal to the mean free path of charged particles can become comparable with their thermal energy. In this case the model of low mo-

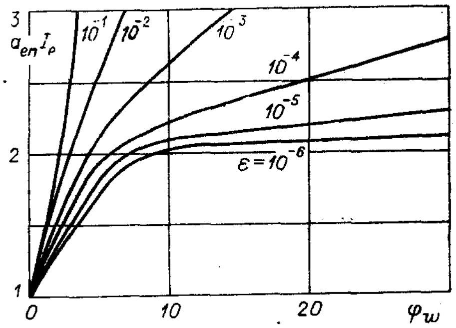

Fig. 1

bility within the Debye layer [11] becomes inapplicable. It can be demonstrated that at electric potentials of the streamlined surface corresponding to the initial ranges of saturation, on the other hand, this model of low mobility within the Debye layer is certainly applicable.

4. We will now extend the analysis in the preceding paragraph to the case of moderate ionization, i.e., will take into account collisions of ions and electrons. In this case the Stefan-Maxwell relations (1.7) become

$$\nabla x_i = \frac{x_i x_e}{\rho} a_{ie}\left(\frac{\mathbf{J}_e}{x_e} - \frac{\mathbf{J}_i}{x_i}\right) - \frac{x_i}{\rho} a_{in}\frac{\mathbf{J}_i}{x_i} - \frac{x_i}{T}\nabla\varphi + x_i\left(\frac{m_i}{m_n} - 1\right)\nabla\ln p + x_i \sum\nolimits_i^{\mathrm{T}} \nabla \ln T,$$
$$\nabla x_e = \frac{x_e x_i}{\rho} a_{ie}\left(\frac{\mathbf{J}_i}{x_i} - \frac{\mathbf{J}_e}{x_e}\right) - \frac{x_e}{\rho} a_{en}\frac{\mathbf{J}_e}{x_e} + \frac{x_e}{T}\nabla\varphi + x_e\left(\frac{m_e}{m_n} - 1\right)\nabla\ln p + x_e \sum\nolimits_e^{\mathrm{T}} \nabla \ln T. \quad (4.1)$$

Adding these equations in the quasineutral approximation again yields Eq. (3.4), and all subsequent arguments in paragraph 3 remain valid. The boundary condition (3.7) still applies, for instance, inasmuch as the first terms on the right-hand sides of Eqs. (4.1) representing collisions between charged particles are negligibly small for the Debye layer and the structure of the latter remains unchanged.

Therefore, in the case of a moderately ionized gas with assumed constant ratios of drag coefficients (ion to neutral particle and electron to neutral particle) one should expect saturation currents according to relations (3.8) and (3.9). The quasineutral concentration of charged particles, moreover, is determined now, as before, according to Eq. (3.5) with boundary conditions (3.6) and (3.7).

5. In deriving the boundary condition (3.7) for the quasineutral concentration of charged particles we have assumed that the thickness of the Debye layer greatly exceeds the local mean free path of charged particles and we have used the hydrodynamic model in [11] for describing this Debye layer. It is quite evident, however, that this boundary condition remains valid also in the case of a collisionless Debye layer. Indeed, the boundary condition for the quasineutral concentration at the streamlined surface is then the condition of balance of attracted particles (electrons when the surface potential is higher than the local plasma potential, ions when the local plasma potential is higher than the surface potential) [12, 13]. This condition is of the order of the local Knudsen number and, therefore, for our case of a sufficiently dense gas we again obtain boundary conditions (3.7).

In this way, the validity of the conclusions arrived at does not depend on whether the Debye layer is a collisional or a collision-free one.

6. The preceding analysis applies to the case where the temperature of electrons is the same as that of heavy particles. In the nonequilibrium case the situation becomes much more complex. Without considering the effects of nonequilibrium in detail, we will only note that sometimes Eq. (3.9) for the ion saturation currents holds approximately true even then. This is so, for instance, in the case of a self-adjoint weakly ionized chemically "frozen" boundary layer with a "frozen" electron temperature, under the condition that the latter is within the nonviscous region equal to the temperature of heavy particles [5].

7. In order to determine the saturation currents according to Eqs. (3.8) and (3.9), it is necessary to first solve the problem (3.5)-(3.7). In the case of a chemically "frozen"

gas stream with a negligible dependence of transport coefficients D_a, Σ_i^T, Σ_e^T on the concentration of charged particles this problem is a linear one. In certain important special cases it can be solved analytically and explicit expressions can be derived for the density of saturation currents. Analytical results can also be obtained for the opposite extreme case of flow under chemical equilibrium.

At high values of the Reynolds number, when the flow field can be subdivided into a nonviscous region and a boundary layer, the concentration of charged particles in the former is constant and equal to x_∞ under conditions of a chemically "frozen" gas flow. It then suffice to solve Eq. (3.5) for the boundary layer only. We note that the solution to the original problem (1.4)-(1.9) for the nonviscous region depends on the structure of its solution for the boundary layer and cannot be obtained independently. In order to construct approximate analytical solutions to Eq. (3.5) for the boundary layer, it is convenient to use the method of successive approximations [14].

In another study [5] there have been derived specific expressions for the density of the ion saturation current flowing through a self-adjoint weakly ionized boundary layer to a wall probe. It would be interesting to compare those expressions with the general expression (3.9) applied to the same conditions.

The expression for the density of the ion saturation current flowing through a self-adjoint chemically "frozen" boundary layer (boundary layer at a flat plate, at a conical surface, and near the critical point on an intensively cooled body) to a wall probe is expression (3.41) in [5]. This expression can be derived from Eq. (3.9) with the aid of the solution [15] to the equation of diffusion for a self-adjoint chemically "frozen" boundary layer.

Expression (3.47) in [5], with a reference to another study [16], describes the density of the ion saturation current into a wall probe at an acute cone in a stream with a high local Mach number and an ionized boundary layer. This expression has been derived on the assumption that convection is negligible when $\eta \leq \eta_m$ [16] (η being a similarity parameter [5] and η_m being the coordinate of the maximum concentration of charged particles). From this assumption follows the relation

$$\left.\frac{\partial x}{\partial \eta}\right|_{\eta=0} = \frac{x(\eta_m)}{\eta_m}.$$

Expression (3.9) can now be reduced to the form (with the notation in [5])

$$N_{em} = 2.55 \cdot 10^{18} \mathrm{Sc}_i \left(\frac{\mathrm{Re}_x}{l}\right)^{1/2} \frac{1}{u_\delta} \frac{\rho_m}{\rho_\delta} \eta_m J_{i,\mathrm{H}}.$$

The structure of this expression is identical to the structure of expression (3.47) in [5], but the numerical coefficient is different (in Eq. (3.47) [5] it is 1.9 rather than 2.55). This discrepancy is due to the fact that the constant on the right-hand side of Eq. (15) in [16] has been assumed to be equal to +2.68, while the asymptotic analysis yields the value +2 for it. This latter value has, by the way, been confirmed by the numerical solutions shown in Fig. 1.

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CONDITIONS FOR SPUTTER EMISSION IN HIGH-PRESSURE SPATIAL GASEOUS DISCHARGES

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The contraction of a high-pressure spatial gaseous discharge is associated with formation of a cathode spot and an outgrowth from the latter of a high-conductance spark channel [1-3]. In an earlier study [4] there was proposed a model of spot initiation under high electric field intensities E(0) at the cathode, when spontaneous emission from individual microasperities becomes significant. Then the cathodic layer is unstable relative to fluctuations of the spontaneous-emission current [5, 6] so that heating of their tips by the electron current and the ion current cause this layer to sputter and a cathode spot is formed. The electric field intensity E(0) is related to the ion current density j according to the law of similitude $E(0)/p = f(j/p^2)$, with p denoting the gas pressure. This relation yields the dependence of the discharge current density on the pressure at a beforehand given electric field intensity $E(0) = E_*$ sufficiently high for initiating a cathodic instability. This study will deal with the determination of the critical electric field intensities E_* and the current densities in spatial discharge at which such intensities are attained.

Calculation of the Electric Field Intensity at the Cathode

In order to find the relation j(p) at a given electric field intensity $E(0) = E_*$, it is necessary to solve the system of nonlinear continuity and Poisson equations

$$-\frac{\partial j_+}{\partial x} = \frac{\partial j_-}{\partial x} = \alpha j_-; \tag{1}$$

$$\frac{dE}{dx} = -\frac{e}{\varepsilon}(n_+ - n_-); \tag{2}$$

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